Obj. 17 Radian Measure

Unit 5 Trigonometric and Circular Functions

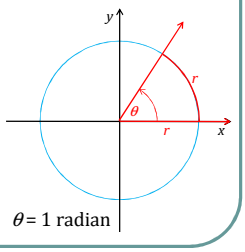
Concepts and Objectives

- Arc Measurement and Rotation (Obj. #17)
  - Convert between degrees and radians
  - Calculate the length of an arc intercepted by a given angle
  - Calculate the area of a sector

Radian Measure

- Up until now, we have measured angles in degrees. Another unit of measure that mathematicians use is called *radian measure*.

An angle with its vertex at the center of a circle that intercepts an arc on the circle equal in length to the radius of the circle has a measure of **1 radian**.



$\theta = 1$ radian

Radian Measure

- You should recall that the circumference of a circle is given by $C = 2\pi r$, where r is the radius of the circle. An angle of 360°, which corresponds to a complete circle, intercepts an arc equal to the circumference.

$$360° = 2\pi \text{ radians}$$
$$180° = \pi \text{ radians}$$
$$1° = \frac{\pi}{180} \text{ radians} \qquad 1 \text{ radian} = \frac{180°}{\pi}$$

If no unit of angle measure is specified, then the angle is understood to be measured in radians.

Radians and Degrees

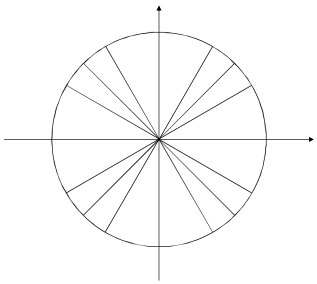
- Converting between radians and degrees is just like converting between any other type of units:
  - Put the unit you are converting *to* on the top, and the unit you are converting *from* on the bottom.
- Example: Convert 120° to radians.

$$\frac{120°}{1}\left|\frac{\pi}{180°}\right. = \frac{120\pi}{180} = \frac{2\pi}{3} \text{ radians}$$

Radians and Degrees

- Example: Convert 57° 48′ to radians
- Example: Convert $\frac{3\pi}{5}$ radians to degrees

The Unit Circle



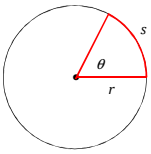
Arc Length of a Circle

- The length of an arc is proportional to the measure of its central angle.

The length s of the arc on a circle of radius r created by a central angle measuring θ is given by

$$s = r\theta$$

(θ must be in radians)



Arc Length of a Circle

- Example: Find the length to the nearest hundredth of a foot of the arc intercepted by the given central angle and radius.

$$r = 1.38 \text{ ft}, \ \theta = \frac{5\pi}{6}$$

Arc Length of a Circle

- Example: Find the length to the nearest tenth of a meter of the arc intercepted by the given central angle and radius.

$$r = 2.9 \text{ m}, \ \theta = 68°$$

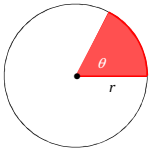
Area of a Sector of a Circle

- Recall that a *sector* is the portion of the interior of the circle intercepted by a central angle. The area of the sector is proportional to the area of the circle.

The area A of a sector of a circle of radius r and central angle θ is given by

$$A = \frac{1}{2}r^2\theta$$

(θ must be in radians)



Area of a Sector of a Circle

- Example: Find the area of a sector of a circle having the given radius r and central angle θ (round to the nearest kilometer).

$$r = 59.8 \text{ km}, \ \theta = \frac{2\pi}{3}$$

Homework

- *College Algebra*
  - Page 565: 25-65 (×5s), 68, 75, 80, 85, 87, 88, 90, 124, 126
- Classwork: *College Algebra*
  - Page 565: 8, 12, 14, 26, 32, 38, 43, 46, 56, 58