Obj. 7 Zeros of Polynomial Functions

Unit 2 Quadratic and Polynomial Functions

Concepts and Objectives

- Zeros of Polynomial Functions (Obj. #7)
- Find rational zeros of a polynomial function
- Use the Fundamental Theorem of Algebra to find a function that satisfies given conditions
- Find all zeros of a polynomial function

Factor Theorem

The polynomial \( x + k \) is a factor of the polynomial \( f(x) \) if and only if \( f(k) = 0 \).

Example: Determine whether \( x + 4 \) is a factor of \( f(x) = x^4 - 48x^2 + 8x + 32 \).

Rational Zeros Theorem

In other words, the numerator is a factor of the constant term and the denominator is a factor of the first coefficient.

Example: For the polynomial function defined by \( f(x) = 8x^4 - 26x^3 - 27x^2 + 11x + 4 \),
(a) List all possible rational zeros
(b) Find all rational zeros and factor \( f(x) \) into linear factors.

Fundamental Theorem of Algebra

Every function defined by a polynomial of degree 1 or more has at least one complex zero.

A function defined by a polynomial of degree \( n \) has at most \( n \) distinct zeros.

The number of times a zero occurs is referred to as the multiplicity of the zero.
Fundamental Theorem of Algebra

- Example: Find a function $f$ defined by a polynomial of degree 3 that satisfies the following conditions.
  (a) Zeros of −3, −2, and 5; $f(−1) = 6$
  (b) 4 is a zero of multiplicity 3; $f(2) = −24$

Conjugate Zeros Theorem

This means that if $3 + 2i$ is a zero for a polynomial function with real coefficients, then it also has $3 - 2i$ as a zero.

Conjugate Zeros Theorem

- Example: Find a polynomial function of least degree having only real coefficients and zeros −4 and $3 - i$.

Putting It All Together

- Example: Find all zeros of $f(x) = x^4 - x^3 - 17x^2 + 55x - 50$ given that $2 + i$ is a zero.

Homework

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