3-2 Zeros of Polynomial Functions

Unit 3 Quadratic and Polynomial Functions

Concepts and Objectives

- Objective #10
  - Find rational zeros of a polynomial function
  - Use the Fundamental Theorem of Algebra to find a function that satisfies given conditions
  - Find all zeros of a polynomial function

Factor Theorem

The polynomial $x - k$ is a factor of the polynomial $f(k)$ if and only if $f(k) = 0$.

Example: Determine whether $x + 4$ is a factor of $f(x) = 3x^4 - 48x^2 + 8x + 32$
Rational Zeros Theorem

If \( \frac{p}{q} \) is a rational number written in lowest terms, and if \( \frac{p}{q} \) is a zero of \( f \), a polynomial function with integer coefficients, then \( p \) is a factor of the constant term and \( q \) is a factor of the leading coefficient.

In other words, the numerator is a factor of the last number and the denominator is a factor of the first coefficient.

Rational Zeros Theorem

- Example: For the polynomial function defined by \( f(x) = -8x^4 - 26x^3 - 27x^2 + 11x + 4 \)  
  (a) List all possible rational zeros  
  (b) Find all rational zeros and factor \( f(x) \) into linear factors.

Fundamental Theorem of Algebra

Every function defined by a polynomial of degree 1 or more has at least one complex zero.

A function defined by a polynomial of degree \( n \) has at most \( n \) distinct zeros.

The number of times a zero occurs is referred to as the multiplicity of the zero.
**Fundamental Theorem of Algebra**

- Example: Find a function $f$ defined by a polynomial of degree 3 that satisfies the following conditions.
  
  (a) Zeros of -3, -2, and 5; $f(-1) = 6$

  (b) 4 is a zero of multiplicity 3; $f(2) = -24$

**Conjugate Zeros Theorem**

If $f(x)$ defines a polynomial function having only real coefficients and if $z = a + bi$ is a zero of $f(x)$, where $a$ and $b$ are real numbers, then $\bar{z} = a - bi$ is also a zero of $f(x)$.

This means that if $3 + 2i$ is a zero for a polynomial function with real coefficients, then it also has $3 - 2i$ as a zero.

**Conjugate Zeros Theorem**

- Example: Find a polynomial function of least degree having only real coefficients and zeros -4 and $3 - i$. 
Putting It All Together

Example: Find all zeros of \( f(x) = x^4 - x^3 - 17x^2 + 55x - 50 \) given that \( 2 + i \) is a zero.