Obj. 19 Graphs of Sine and Cosine

Unit 5 Trigonometric and Circular Functions

Concepts and Objectives

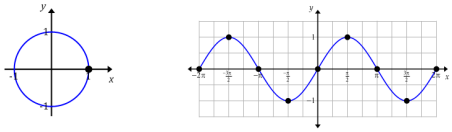
- Graphs of the Sine and Cosine Functions (Obj. #19)
  - Be able to graph the sine and cosine functions on the interval $[-2\pi, 2\pi]$
  - Be able to identify how the graphs of the sine and cosine change due to changes in
    - Amplitude
    - Period
    - Vertical translation
    - Phase shift

Graphing the Sine Function

- Because the sine is a *circular* function, while θ can be any number (positive or negative), the value of sin θ only goes from –1 to 1. (It is also considered a *periodic* function.)
- To graph a circular function, we can take the angle as our *x*-coordinate and the corresponding coordinates from the unit circle as our *y*-coordinate.

Graphing the Sine Function

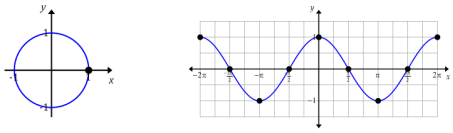
- If we graph these values on a coordinate plane, we can see how this function curves:



- The key to graphing the sine function is that it includes the point (0, 0).

Graphing the Cosine Function

- The cosine function is very similar to the sine. Let's look at a graph of it:



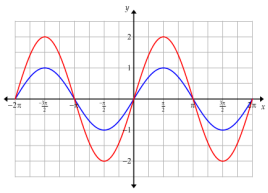
- The key to graphing the cosine function is that it includes the point (0, 1).

Changing the Basic Graph

- *Amplitude* describes how the basic graph is changed vertically (taller or shorter). The amplitude of a periodic function is half the difference between the maximum and minimum values.
- *Period* describes the "cycle" of the graph—how long it takes for the graph to start repeating.
  - The period of the sine or cosine is 2π.

Amplitude

- Compare the graphs of $y = \sin x$ and $y = 2\sin x$



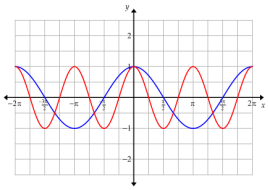
- Notice that both graphs cross the x-axis at the same places and that both graphs peak at the same places.

Amplitude

- The graph of $y = a \sin x$ or $y = a \cos x$, with $a \neq 0$, will have the same shape as the graph of $y = \sin x$ or $y = \cos x$, respectively, except with the range $[-|a|, |a|]$. The amplitude is $|a|$.
- Example: What is the amplitude of $y = \frac{1}{3}\cos x$? What is the range of the graph?

Period

- Compare the graphs of $y = \cos x$ and $y = \cos 2x$.



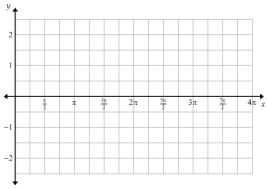
- The two graphs have the same height, but the second graph is "compressed".

Period

- For $b > 0$, the graph of $y = \sin bx$ will resemble that of $y = \sin x$, but with period $\frac{2\pi}{b}$. Also, the graph of $y = \cos bx$ will resemble that of $y = \cos x$, but with period $\frac{2\pi}{b}$
- To calculate the period of a function, substitute the coefficient of x in for b and reduce.

Sketching a Circular Graph

- Example: Sketch the graph of $y = -2\sin\frac{x}{2}$ over one period.



Translating Sine and Cosine

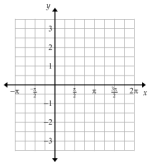
- We have seen what the graph of $y = a \sin bx$ looks like. Next, we can shift the graph vertically and/or horizontally.
- The full form of the sine function is
$$y = c + a\sin b(x - d)$$
  - c affects the vertical position of the graph. A positive c shifts the graph c units up, and a negative c shifts the graph c units down.
  - d shifts the graph horizontally. $(x + d)$ shifts the graph d units to the **left**, and $(x - d)$ shifts the graph d units to the **right**.

Translating Sine and Cosine

- With circular functions, a horizontal translation is called a *phase shift*. The phase shift is the absolute value of *d*.
- To sketch the translated graph, you can either divide the interval into four parts (eight parts for two periods) and chart the values as before, or you can sketch the stretched/compressed parent graph and translate it according to *c* and *d*.
  - The second method is probably the easiest to do once you are comfortable with the basic graphs.

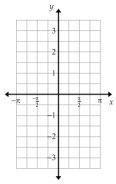
Graphing Sine and Cosine

- Example: Graph $y = 3\cos\left(x + \frac{\pi}{4}\right)$ over one period.



Graphing Sine and Cosine

- Example: Graph $y = -1 + 2\sin(4x + \pi)$ over two periods.



Homework

- *College Algebra*
  - Page 593: 15, 16, 19, 23, 34
    - HW: 16, 20, 34, 42, 44